



Fluid Dynamics and Transport Phenomena

Laminar natural convection characteristics in an enclosure with heated hexagonal block for non-Newtonian power law fluids



Krunal M. Gangawane*, B. Manikandan

Department of Chemical Engineering, College of Engineering Studies, University of Petroleum and Energy Studies, Dehradun 248007, Uttarakhand, India

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ABSTRACT

This work illustrates the steady state, two dimensional natural convective flow and heat transfer features in square enclosure containing heated hexagonal block maintained either at constant wall temperature (CWT) or uniform heat flux (UHF) thermal conditions. Governing equations (mass, momentum and energy) are solved by using finite volume method (FVM) with 3rd order accurate QUICK discretization scheme and SIMPLE algorithm for range of field pertinent parameters such as, Grashof number ($10^3 \leq Gr \leq 10^6$), Prandtl number ($1 \leq Pr \leq 100$) and power law index ($0.5 \leq n \leq 1.5$). The analysis of momentum and heat transfer characteristics are delineated by evolution of streamlines, isotherms, variation of average Nusselt number value and Colburn factor for natural convection (j_{nH}). A remarkable change is observed on fluid flow and thermal distribution pattern in cavity for both thermal conditions. Nusselt number shows linear variation with Grashof and Prandtl numbers; while rate of heat transfer by convection decreases for power law index value. Higher heat transfer rate can be achieved by using uniform heat flux condition. A Nusselt number correlation is developed for possible utilization in engineering/scientific design purpose.

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1. Introduction

The analysis of heat transfer processes along with mechanics of non-Newtonian fluids is of great significance due to its wide range of applications in nature. Many fluids which quite frequently observed in various industries exhibit non-Newtonian behavior, such as, shear-thinning (Pseudoplastic), visco-elastic, shear-thickening (dilatant), *etc.* under suitable concentrations and/or flow conditions. Such materials include oil–water emulsions, froths and foams, sewage sludge, butter, gas liquid dispersions, biological fluids, paints, foodstuffs and many more [1,2]. Moreover, it has also been acknowledged that, in day to day activities, fluids with non-Newtonian behavior are quite frequently encountered than Newtonian one. Non-Newtonian fluids differ from Newtonian one due to its non-linear dependence of shear stress with shear rate. As the dynamic viscosity varies with shear stress, its flow characteristics and thereby heat transfer become more complex [1,2]. In order to delineate the non-Newtonian fluid behavior, several mathematical models are present (such as, power-law model, Carreau–Yesuda model, *etc.*). Out of all these models, the power law (or Ostwald de Waele) model has gained much popularity due to simplicity along with adequate representation of fluids with broad range of shear rates [3].

Owing to its fundamental and pragmatic significance, natural convection in square/rectangular enclosures has been extensively studied and it is considered as one of the largely invaded area of heat transfer. The reason for overwhelming popularity of heat transfer in enclosures (closed as well open) configurations is due to simple structure with ability to explore broad varieties of fluid flow and heat transfer fundamentals (boundary layer, vortex size and location, circulation of fluid, *etc.*). Typical examples which can be idealized as enclosures are, for instance, cooling systems of electronic gadgets, high performance building insulation, multi-shed structures, furnace, food processing (heating of various food stuffs like beans, carrot and potato chips, *etc.*), lubrication technologies, fluidized bed drying of fibrous substances, solar heat collectors, drying, *etc.* [4–11].

On the other hand, detached obstacle of varying shapes (circular, square, triangular, *etc.*) is used for controlling fluid flow due to convection [12] in enclosure, which is needed for applications where fluid flow due to convection should be restricted or bifurcated (crystal growth, solidification, *etc.*). Though many studies have revealed the natural/forced/mixed convection characteristics in enclosure with adiabatic/isothermal obstacle of circular/square shape [12–20], no information, as much known to authors, is available for convection characteristics from hexagonal block. Moreover, the natural/forced or combined convection from heated objects in enclosures have practical findings such as, heat exchangers [21]. Further, it is also known that the square block is more bluff body than circular one. The basic difference in fluid flow structure occurs due to the presence of cornered

* Corresponding author.

E-mail addresses: krunalgangawane@gmail.com, kmgangawane@ddn.upes.ac.in (K.M. Gangawane).

edges of block [22]. Therefore, the presence of six, slightly inclined edges in hexagonal block can significantly affect momentum and heat transfer characteristics as compared to block of other shape.

Additionally, in heat transfer studies, we come across various thermal boundary conditions, for example, constant temperature, heat flux, linear, sinusoidal, etc. These thermal conditions have remarkable influence on fluid flow and heat transfer characteristics in enclosures (due to change in boundary layer structure). As compared to natural or mixed convection characteristics from heated obstacles in cavity/enclosures, large amount of studies pertain to constant temperature condition. Very modest is known about the effect of other important thermal conditions (linear, heat flux, etc.) on the natural convection characteristics in enclosure. Therefore, in present work, difference in flow pattern and heat transfer in enclosure is analyzed for heated hexagonal block with either of two limiting thermal conditions of constant wall temperature (CWT) and uniform heat flux (UHF). It constitutes the main objective of the work, i.e., exploring momentum and thermal transfer characteristics in an enclosure with centrally placed heated hexagonal block. But before presenting the detailed problem formulation and new simulation results, it is practicable to present the selected previous literature pertaining to natural/mixed convection in enclosures with/without heated body/block for Newtonian as well non-Newtonian fluids.

2. Previous Work

Literature covering heat transfer aspects in enclosures with heated blocks is large for Newtonian fluids [12–20]. Many studies have already explored the influence of heated/adiabatic circular [21–33], sinusoidal [34], elliptical [35], triangular [36–39], square [12,40] blocks on natural/mixed convection characteristics for Newtonian fluids. However, limited information is available on natural convection characteristics for non-Newtonian fluids in enclosure with/without detached bodies of varying shapes and thermal conditions. For instance, Hussain and Hussein [25] investigated detailed convection characteristics due to buoyancy in cavity containing circular cylinder with uniform heat flux boundary condition. Lamsaadi *et al.* [41] presented the natural convection for non-Newtonian power law fluids in rectangular shallow cavity by numerical as well as analytical approach. They developed theoretical solution on the basis of non-Newtonian fluids. Cheng [42] reported influence of mixed thermal boundary condition (heat flux and temperature) on natural convection characteristics in a porous media from a vertical cone for power law fluids. Lower values of surface heat flux and temperature are obtained for shear thickening fluids. Few studies also have delineated the Bingham fluid flow features due to natural/mixed convection [43–46]. In addition to that, Turan *et al.* [47] studied on natural convection analysis in differentially heated cavity for non-Newtonian power law fluids for range of governing parameters (Rayleigh number, Prandtl number, power law index, etc.). The average Nu approaches unity for shear thickening fluids. Whereas, Khalifa *et al.* [48] chose horizontal porous cavity as computational model for analyzing non-Newtonian power law behavior for range of governing parameters. They predicted criteria for sub and supercritical onset of fluid flow. Sasmal and Chhabra [49] explored momentum and heat transfer characteristics due to natural convection from heated square cylinder. For numerical experimentation, they used range of parameters, such as, Grashof number ($10 \leq Gr \leq 10^5$), Prandtl number ($0.72 \leq Pr \leq 100$) and power law index ($0.3 \leq n \leq 1.8$). For otherwise similar conditions, shear thinning fluids showed higher heat transfer rate. Similar study is conducted by Chandra and Chhabra [50] for another shape of obstacle/block, i.e., semi-circular cylinder. Subsequently, Matin *et al.* [51] studied natural convection for non-Newtonian power law fluids between two-square shaped eccentric duct annuli. There numerical study was focused on effect of eccentricity for range of governing parameters. They reported negligible effect of Prandtl number on averaged Nu variation.

On the other hand, numerical study of double diffusive natural convection for Newtonian fluid in cavity with heated square body is reported by Nazari *et al.* [52] by using kinetic, lattice Boltzmann method. The results of the study shown increase in average Nu with Rayleigh and Sherwood numbers. Selimefendigil and Oztop [53] shown comparison of effect of shape of obstacle placed inside cavity on flow structure due to magneto-hydrodynamics (MHD) natural convection. They investigated the effect of different shapes (circular, square and diamond) on convection characteristics in cavity for nanofluids. They reported reduction of average Nu values due to the presence of obstacle by 21.35% (circular), 32.85% (diamond) and 34.64% (square) as compared to cavity without block. Recently, Ren and Chan [54,55] elucidated the influence of array of square blocks on natural convection characteristics by using lattice Boltzmann method and CUDA platform. It is observed that GPU can enhance the computation speed by a factor up to 20 as compared to the non-parallel CPU code.

Therefore, from review of available literature, it can be said that immense knowledge is available for various aspects of hydrodynamics and heat transfer behavior in cavity with heated block for Newtonian fluids [23–40]. Much less is known for non-Newtonian fluid flow characteristics in cavity with detached heated body for different thermal conditions. Few studies have delineated natural convection characteristics for non-Newtonian fluids in cavity with square/circular body [50,51]. No information, as much know to authors, is available on natural convection characteristics from heated hexagonal block detached in enclosure for different thermal conditions and power law fluids. The present study aims to fulfill the discrepancy found in literature. In particular, the natural convection heat transfer characteristics from heated hexagonal block have been explored for range of Grashof number ($10^3 \leq Gr \leq 10^6$), Prandtl number ($1 \leq Pr \leq 100$) for power law index ($0.5 \leq n \leq 1.5$) for two limiting thermal conditions of constant wall temperature (CWT) and uniform heat flux (UHF).

3. Problem Statement and Mathematical Formulation

The schematic of geometry and coordinate system is depicted in Fig. 1. Geometry of problem consists of square enclosure containing centrally placed hexagonal block. The cavity contains fluid which is assumed to be steady, incompressible and obeys non-Newtonian power law. Hexagonal block is maintained either at constant wall temperature, T_w , (CWT) or uniform heat flux, q , (UHF). Vertical walls of cavity are exposed to ambient temperature ($T_c < T_w$), while horizontal

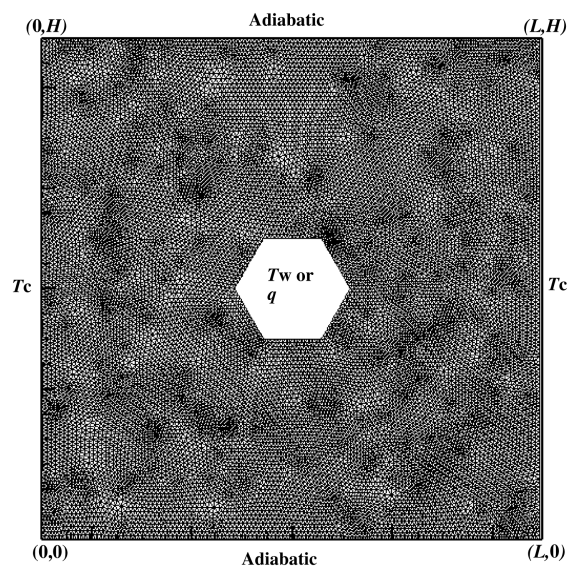


Fig. 1. Schematics of physical as well as computational domain for problem under consideration along with mesh structure.

walls are maintained at adiabatic condition. All walls of cavity are maintained at no-slip condition. Further, the thermo-physical properties of the working fluid are assumed to be independent of the temperature and the viscous dissipation effects as well as radiation heat transfer are assumed to be neglected. The extent of the variation of density with temperature is expressed by using Boussinesq approximation. Hence, the mass, momentum and energy equations for natural convection in non-dimensional form are expressed as follow [50].

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0 \quad (1)$$

$$u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{\sqrt{Gr}} \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \right) \quad (2)$$

$$u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{\sqrt{Gr}} \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} \right) + \theta \quad (3)$$

τ_{ij} represents shear stress tensor. For fluids obeying non-Newtonian power law (or Ostwald-De Waele) model, it is expressed as,

$$\tau_{ij} = 2\eta D_{ij} = \eta \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (4)$$

where, D_{ij} represents the rate of deformation tensor for the two dimensional Cartesian coordinate and η is apparent viscosity which is also expressed as,

$$\eta = m \left(2 \left[\left(\frac{\partial u_x}{\partial x} \right)^2 + \left(\frac{\partial u_y}{\partial y} \right)^2 \right] + \left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y} \right)^2 \right)^{\frac{(n-1)}{2}} \quad (5)$$

Above expression contains, m and n which are consistency index and power law index, respectively. For $n = 1$, Eq. (5) simplifies to Newtonian fluids. The general energy equation for two-dimensional flow is expressed as follows:

$$u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y} = \frac{1}{Pr \cdot Gr^{\frac{1}{n+1}}} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (6)$$

Non-dimensional equations for natural convection are obtained by following mathematical rearrangements of flow parameters,

$$\begin{aligned} x &= \frac{x^*}{H}; y^* = \frac{y^*}{H}; \\ u_x &= \frac{u_x^*}{V}; u_y = \frac{u_y^*}{V}; \\ V &= \sqrt{g_y \beta \Delta T H} \\ \text{CWT : } T &= \frac{\theta - \theta_\infty}{\theta_w - \theta_\infty}; \text{UHF : } \theta = \frac{\theta - \theta_\infty}{\frac{H}{q k}}; \\ p &= \frac{H^2 p^*}{\rho \alpha^2} \end{aligned} \quad (7)$$

where, arched (*) variables are dimensional one. The consistent boundary conditions for the problem under consideration (Fig. 1) are expressed in non-dimensional form as follows:

- West (0,y) and East (1,y) walls: These have no-slip walls and are exposed to ambient isothermal temperature (T_c).

$$u_x = u_y = 0; T_c = 0; \text{ for } 0 \leq y \leq 1 \text{ at } x = 0 \text{ or } 1 \quad (8)$$

- Top (x,1) and bottom walls (x,0): It has no-slip wall and was maintained at adiabatic condition.

$$u_x = u_y = 0; \frac{\partial T}{\partial y} = 0 \text{ for } 0 \leq x \leq 1 \text{ at } y = 0 \text{ or } 1 \quad (9)$$

- Hexagonal block: The centrally placed block is stationary and maintained either at CWT or UHF conditions.

$$\text{CWT : } u_x = u_y = 0 \quad T = 1 \quad (10)$$

$$\text{UHF : } u_x = u_y = 0 \quad \frac{\partial T}{\partial n} = -1 \quad (11)$$

The dimensionless groups (for non-Newtonian fluids) appearing in above equations are represented as follows [50]:

- Prandtl number (Pr) = $\frac{C_p \rho}{k} \left[\frac{m}{\rho} \right]^{\frac{(2-n)}{(1+n)}} H^{\frac{(1-n)}{(1+n)}} [H \rho g_y \beta \Delta T]^{\frac{(3(n-1))}{(2(n+1))}}$
- Grashof number (Gr) = $\frac{\rho^2 H^{n+2} (g_y \beta \Delta T)^{2-n}}{m^2}$

The numerical simulation of governing field equations (Eqs. (1)–(6)) in conjunction with the above noted boundary conditions (Eqs. (8)–(11)) yields the primitive variable fields, such as, velocity, pressure, temperature, etc. These parameters are further utilized and processed to obtain the engineering parameter, such as, streamlines, vorticity, Nusselt number, etc. These physical characteristics can be obtained as follows:

- Stream function (ψ): It is estimated from velocity field as,

$$\psi = \int u_x dy \quad (12)$$

- Vorticity (ϕ) or tendency to rotate at point is expressed as,

$$\phi = \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} \quad (13)$$

- Nusselt number: The local Nusselt number on the surface of the heated hexagonal block is estimated by the following expressions:

$$\begin{aligned} \text{CWT : } Nu &= \frac{hH}{k} = -\frac{\partial T}{\partial s} \\ \text{UHF : } Nu &= \frac{hH}{k} = \frac{1}{T} \end{aligned} \quad (14)$$

where, s is normal drawn to the edge of hexagonal block. The obtained local values have been further averaged over the surface of a block to estimate the surface averaged Nusselt number. It is expressed as follows [57]:

$$\overline{Nu} = \int (Nu) ds \quad (15)$$

It is known from dimensional analysis that the average Nusselt number is a function of the Grashof number, Prandtl number, and type of thermal boundary condition for the problem studied herein [57]. The selection of simulation parameters (i.e., grid, domain size, numerical tool, discretization type, etc.) has been discussed in subsequent section.

4. Procedure for Numerical Simulation

Governing partial differential equations (mass, momentum and energy) are discretized by using finite volume method (FVM). The solution of these equations has been obtained by using the ANSYS FLUENT (version 14) commercial CFD solver. The unstructured 'triangular' cells of non-uniform grid spacing have been generated as shown in Fig. 1. Furthermore, well-known semi-implicit method for the pressure linked equations (SIMPLE) scheme is used to solve the pressure-velocity decoupling. The set of algebraic equations are further solved by using the Gauss-Siedel (G-S) point-by-point iterative scheme. The momentum and energy terms are discretized by means of third order accurate

QUICK (Quadratic Upwind Interpolation for Convection Kinetics). It uses 3-point to point upstream weighted quadratic interpolation for calculation of cell face values. The details of QUICK scheme can be found in [56]. Iterative process is terminated for convergence criteria of 10^{-7} based on the normalized residuals for each field equation. For general variable of ζ , the generalized convergence equation at node 'M' is expressed as follows [32]:

$$\Gamma^M \zeta^M = \sum_{NM} \Gamma^{NM} \zeta^{NM} + \lambda \quad (16)$$

Further, the scaled residual can be given as below.

$$R^f = \frac{\sum_{\text{Cells}} \left| \sum_{N\lambda} \Gamma^{NB} \zeta^{NB} + \lambda - \Gamma^M \zeta^M \right|}{\sum_{\text{Cells}} \Gamma^M \zeta^M} \quad (17)$$

where, M , N , Γ^{NB} and B are node, neighboring node, influence coefficients for the neighboring nodes and constant part, respectively. Moreover, under relaxation of variables is performed in order to avoid divergence. These factors are maintained at 0.7, 0.3 and 0.7 for velocity, pressure and temperature, respectively. In numerical simulation studies, proper choice of grid and domain sizes is a very important step as the accuracy of results is dependent on these parameters. Therefore, prudent choice of these parameters is a must. Next section examines grid independence study as well as validation of present numerical procedure.

4.1. Choice of numerical parameters and validation

The dependability as well as accuracy of the numerical simulation procedure is naturally dependent upon a right choice of optimal parameters such as sizes of the computational domain and computational grid. In this work, the size of computational domain is itself delineated by the problem; therefore, a thorough grid independence study has been carried out using five non-uniform of triangular grids (G1, G2, G3, G4 and G5). The details of grid size showing number of nodes and elements are given in Table 1. The influence of different grid sizes on average Nusselt number values is shown in Fig. 2. It is observed that minimum and maximum deviations between G1 and G5 is 22% ($Gr = 10^3$,

Table 1
Different grid structures (number of nodes and elements) utilized for grid independence study

Grid	G1	G2	G3	G4	G5
Nodes	5002	12220	18200	22568	25986
Elements	8164	21567	35776	43765	51678

$n = 0.5$) and 13% ($Gr = 10^3$, $n = 1.5$), respectively. The average difference between G1 and G5 is estimated to be 2.4%. On the other hand, if deviation between G3 and G5 is concerned, maximum and minimum differences reduce to 5.8% ($Gr = 10^3$, $n = 1.5$) and 3.2% ($Gr = 10^3$, $n = 0.5$), respectively. It can also be clearly seen (Fig. 2) that the effect of change in grid size after G3 is insignificant with enormous increase in the computational time as well as memory. Further, in changing grid size after G3, the computational time increases by almost two-three times.

Thus, keeping in mind the factors such as, accuracy, computational time and space, triangular grid G3 (nodes: 18200; elements: 35776) are found to be optimum and have been used for obtaining results.

After grid independence study, it is required to verify the numerical simulation procedure by comparing obtained results with literature (either of numerical or experimental study). In this work, benchmark problem of differentially heated square cavity is used for validation by comparing average Nusselt number of heated wall with that of previous numerical studies [7,58]. A triangular mesh of non-uniform spacing (nodes: 3445; elements: 6656) is generated for simulation purpose. Numerical simulations are performed for $Ra = 10^3$, 10^4 , 10^5 and 10^6 for $Pr = 0.71$. Table 2 compares the $\bar{Nu}$ values obtained in present simulation with previous studies [7,58]. It is clearly evident from Table 2 that present simulation results show excellent likeness with previous works. The average error is found to be <1%.

Table 2

Validation of numerical procedure based on average Nusselt number of heated wall of differentially heated cavity at $Pr = 0.71$ and $n = 1$

Source	$Ra = 10^3$	$Ra = 10^4$	$Ra = 10^5$	$Ra = 10^6$
Present	1.117	2.246	4.533	8.900
Gangawane <i>et al.</i> [7]	–	2.267	–	8.808
G. Vahl Davis [58]	1.118	2.243	4.519	8.798

In order to ascertain the accuracy of present work, additional simulations are performed for problem of square cavity with centrally placed heated square block [51] for non-Newtonian power law fluids based on average Nusselt number of block. Table 3 shows comparison of simulated Nu values with that of [51] for Rayleigh number (10^4 , 10^5); power law index ($n = 0.6$ and 1.4) for $Pr = 10$. The analysis of Table 3 reveals that maximum and minimum deviations are about 4.3% and 3.2%, for $Ra = 10^4$, $n = 1.4$ and $Ra = 10^5$, $n = 0.6$, respectively. An average error is estimated to be 0.45%. Such discrepancies in the numerical results are quite common in modeling studies. The factors responsible for such deviations are numerical method, grid size and structure,

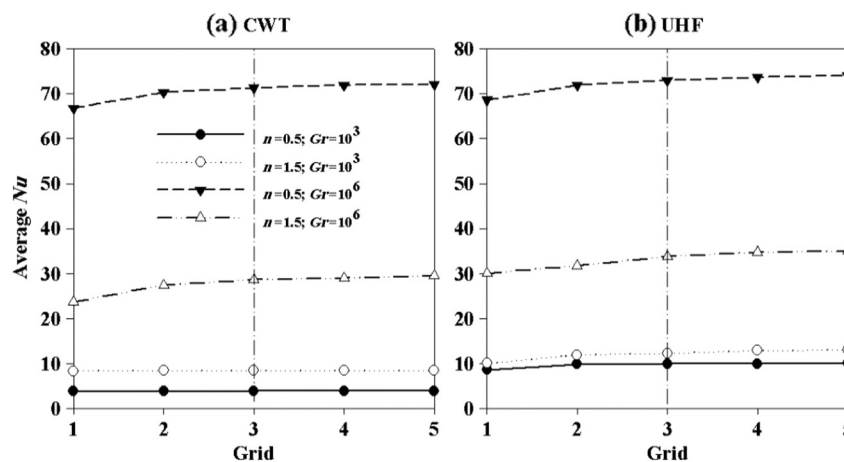


Fig. 2. Variation of average Nu for various grid structures at $Gr = 10^3$; 10^6 power law index of $n = 0.5$, 1.5 and $Pr = 100$ for CWT and UHF thermal conditions.

Table 3

Comparison of average Nu values between present results with that of Martin *et al.*'s [51] based on problem of cavity with centrally placed heated square block for different Ra and power law index values at $Pr = 10$

Source	$Ra = 10^4$		$Ra = 10^5$	
	$n = 0.6$	$n = 1.4$	$n = 0.6$	$n = 1.4$
Present	6.86	4.85	14.83	4.92
Martin <i>et al.</i> [51]	7.01	4.70	15.61	4.85

convergence criteria, *etc.* Hence, minding the above mentioned inadvertent factors affecting the accuracy of numerical results, the validation presented in Tables 2,3 ascertains the confidence in the accuracy and reliability of the present numerical simulation procedure. The results presented herein are, therefore, believed to be accurate and reliable within $\pm (3\%–4\%)$. Having gained the assurance in the accuracy for the present numerical simulation procedure, the subsequent sections present the influence of flow governing parameters (*i.e.*, Prandtl number, Grashof number, power law index, type of boundary condition, *etc.*) on the detailed natural convection heat transfer square cavity containing centrally placed heated hexagonal block in terms of the streamline and isotherm patterns and average Nusselt numbers.

5. Results and Discussion

In this numerical work, the characteristics of natural convection in square cavity with centrally placed hexagonal block have been explored. The dependency of Nusselt number (dimensionless heat transfer coefficient) on governing parameters, such as, Grashof number (Gr), Prandtl number (Pr) and power law index (n) has been elucidated. In particular, the influence of range of various governing parameters that have been used for numerical experiments is given as follows:

Grashof number, $Gr = 10^3, 10^4, 10^5$ and 10^6 ;

Prandtl number, $Pr = 1, 50$ and 100 ;

Power law index, $n = 0.5, 1$ and 1.5 ;

Thermal conditions: constant wall temperature and uniform heat flux (or heat source).

Extensive results are obtained for above mentioned flow parameters revealing the local and global flow and heat transfer characteristics (such as streamline, isotherm contours, average Nusselt number and Colburn factor of heat transfer (j_{nH}) *etc.*) are presented and discussed herein the ensuing section. Further, for contour plots, the numbers of contour lines are kept constant at 15.

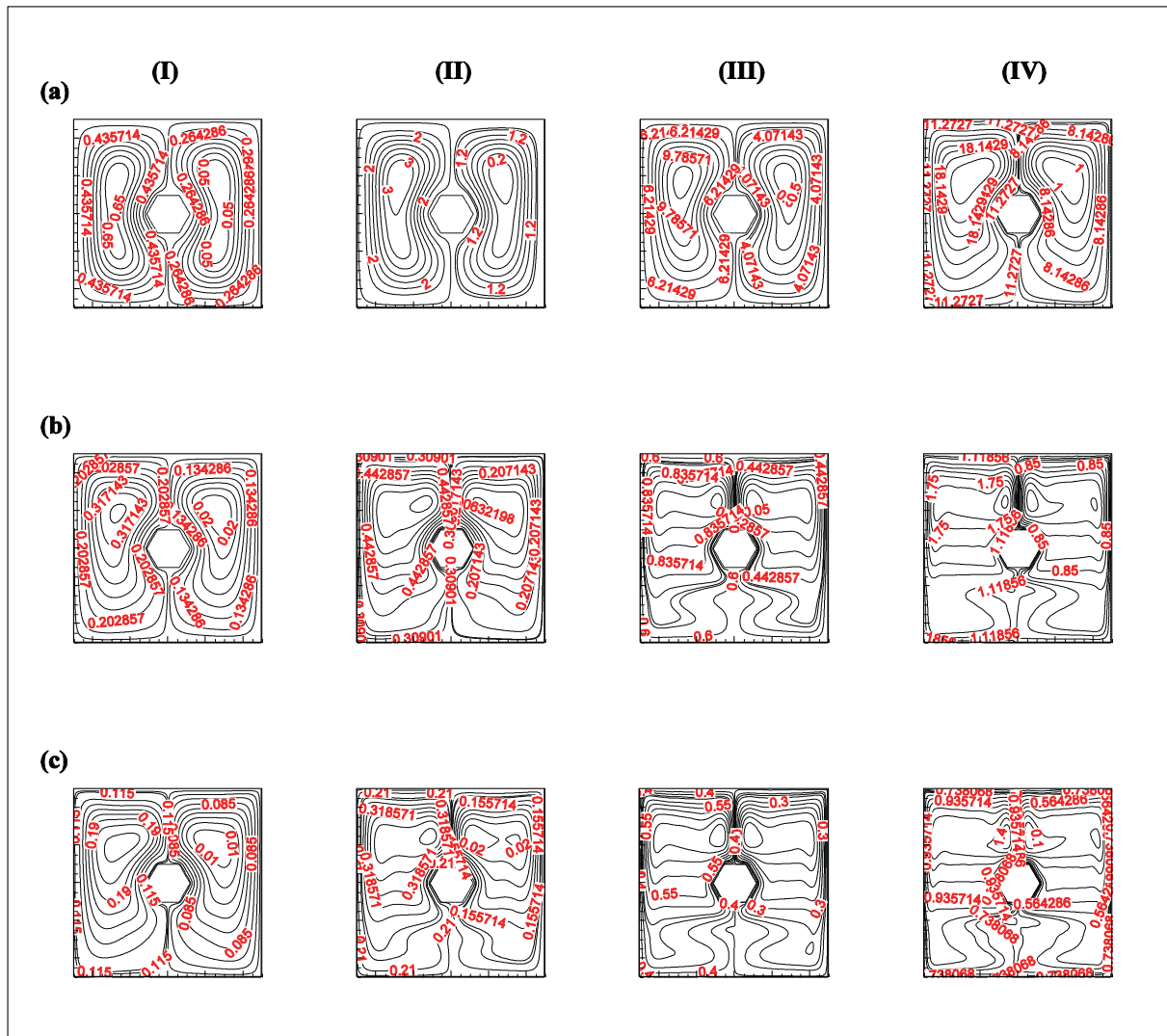


Fig. 3. Variation of streamlines with Grashof ($Gr = \text{I-}10^3$; $\text{II-}10^4$; $\text{III-}10^5$; $\text{IV-}10^6$) and Prandtl numbers ($Pr = \text{a-}1$; $\text{b-}50$ and $\text{c-}100$) for power law index of $n = 0.5$ for constant wall temperature (CWT) condition.

5.1. Streamline and isotherm patterns

It is widely acknowledged that the flow and heat transfer from heated body/wall is affected mainly due to thickness of boundary layers, i.e., momentum as well as thermal boundary layers. In natural convection studies, the parameters which influence these boundary layers are mainly, Grashof number, Prandtl number and power law index (in case of non-Newtonian fluids) [7]. Therefore, these dependencies have been explored in ensuing paragraphs.

Figs. 3–8 depict the streamline structures for range of Grashof numbers (I- 10^3 ; II- 10^4 ; III- 10^5 ; IV- 10^6) corresponding to laminar flow and Prandtl numbers (a-1; b-50; c-100) for thermal conditions of CWT (Figs. 3–5) followed by UHF (Figs. 6–8) for power law index in increasing order, i.e., $n = 0.5$, 1 and 1.5. It is obvious from these figures that the variations are largely due to Prandtl number followed by Grashof number and then power law index. In general, fluid near the vicinity of heated block experiences decrease in density due to rise in temperature. These lighter fluid elements then move in upward direction and strike the top wall (which is adiabatic). Fluid elements, then, start approaching cold vertical wall with gradual decrease in the temperature. This again accelerates the density of fluid, making it heavier. The fluid then settles at the bottom wall. These settled cold fluid elements then again approach the heated hexagonal body with gradual

augmentation in temperature, thus completing the fluid circulation/rotation. As the fluid circulation remains mostly close the active walls (i.e., from surface of block to front vertical wall), the core region of circulation remains nearly stagnant. This quasi-motionless region formed in the core of circulation can be named as convection cell or vortex. Furthermore, the flow is observed to be symmetric about the vertical axis, on either sides of heated block. Such behavior is quite common in the study of cavity with heated block. Such symmetric/bifurcated flow around vertical block are reported for other shapes (circular, square, etc.) also [12,25,32,55]. Moreover, Grashof number is associated with the strength of buoyancy driven flow. The influence of increase in Grashof number ($1-10^3$; II- 10^4 ; III- 10^5 ; IV- 10^6) can be clearly seen on the structure of streamlines (Figs. 3–8). The significant variation in the shape of convection cell is observed with Gr for otherwise identical conditions. The increase in Gr promotes rapid circulation of fluid between active walls (i.e., hot and cold surfaces). This rapid circulation stretches the convection cell and pulls it vertically upward, clearly indicating rise in natural convection phenomenon. Prandtl number shows significant effect on overall flow structure in cavity. The rise in Pr causes shrinking of the convection cells and also shifting towards the upper half of cavity. The dominant viscous forces slightly disturb the symmetric of flow due to thickened hydrodynamic boundary layer, which controls the flow circulation near heated hexagonal block. In addition to this, formation

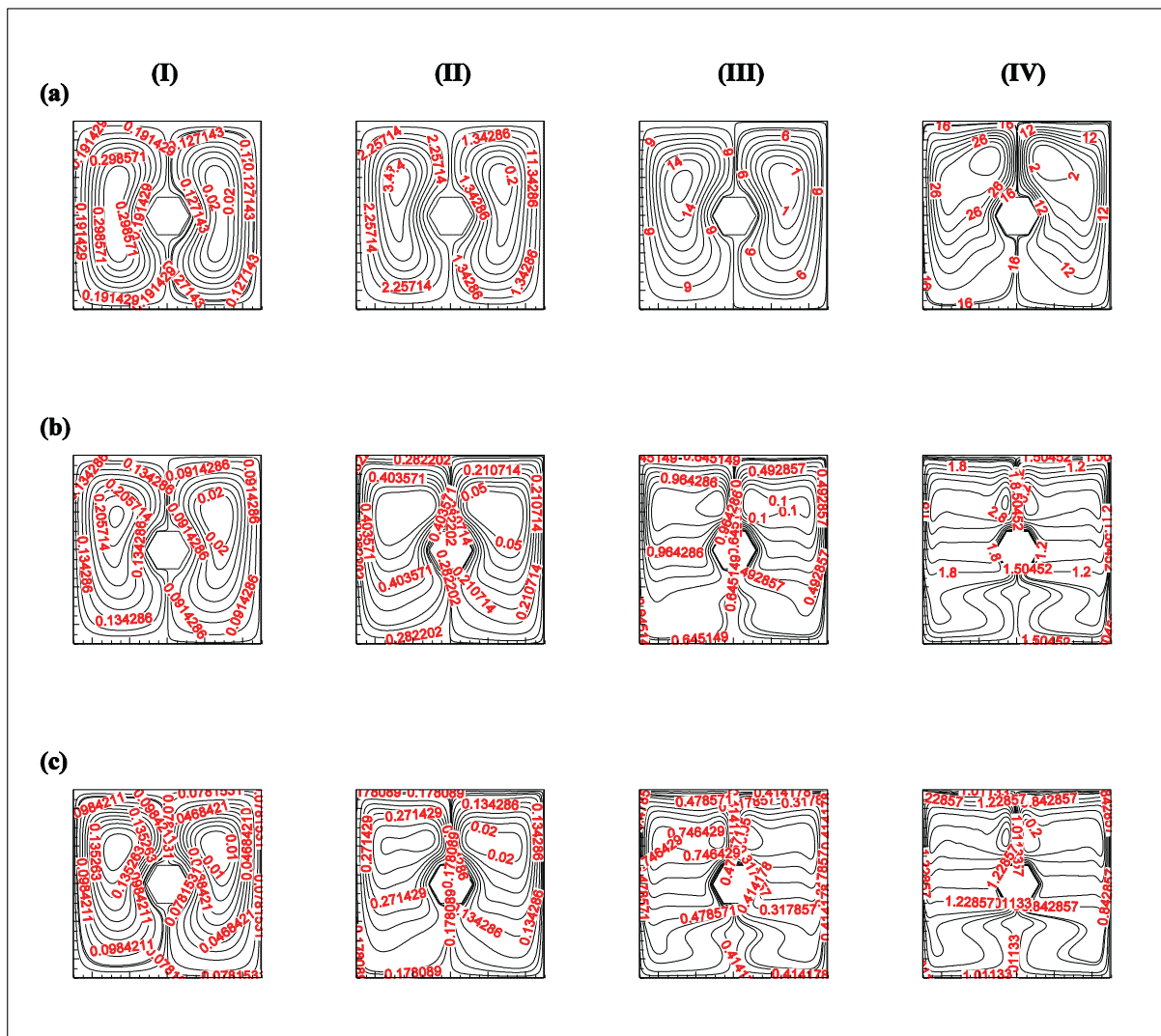


Fig. 4. Variation of streamlines with Grashof ($Gr = 1-10^3$; II- 10^4 ; III- 10^5 ; IV- 10^6) and Prandtl numbers ($Pr = a-1$; b-50 and c-100) for power law index of $n = 1.0$ for constant wall temperature (CWT) condition.

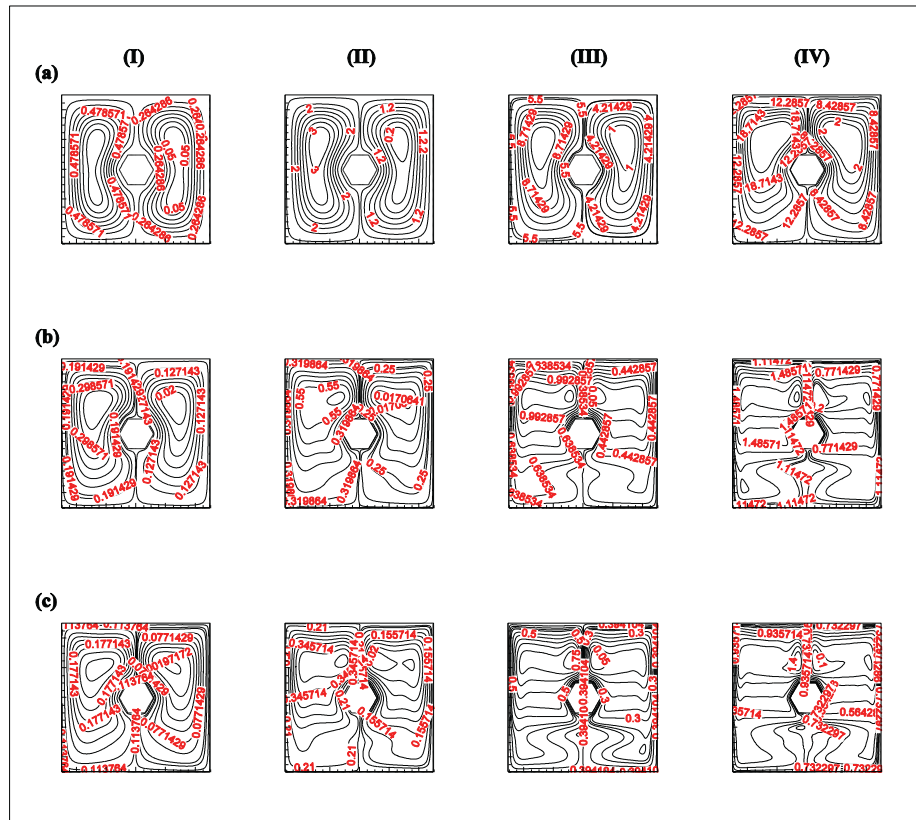


Fig. 5. Variation of streamlines with Grashof ($Gr = \text{I}-10^3$; II- 10^4 ; III- 10^5 ; IV- 10^6) and Prandtl numbers ($Pr = \text{a}-1$; b-50 and c-100) for power law index of $n = 1.5$ for constant wall temperature (CWT) condition.

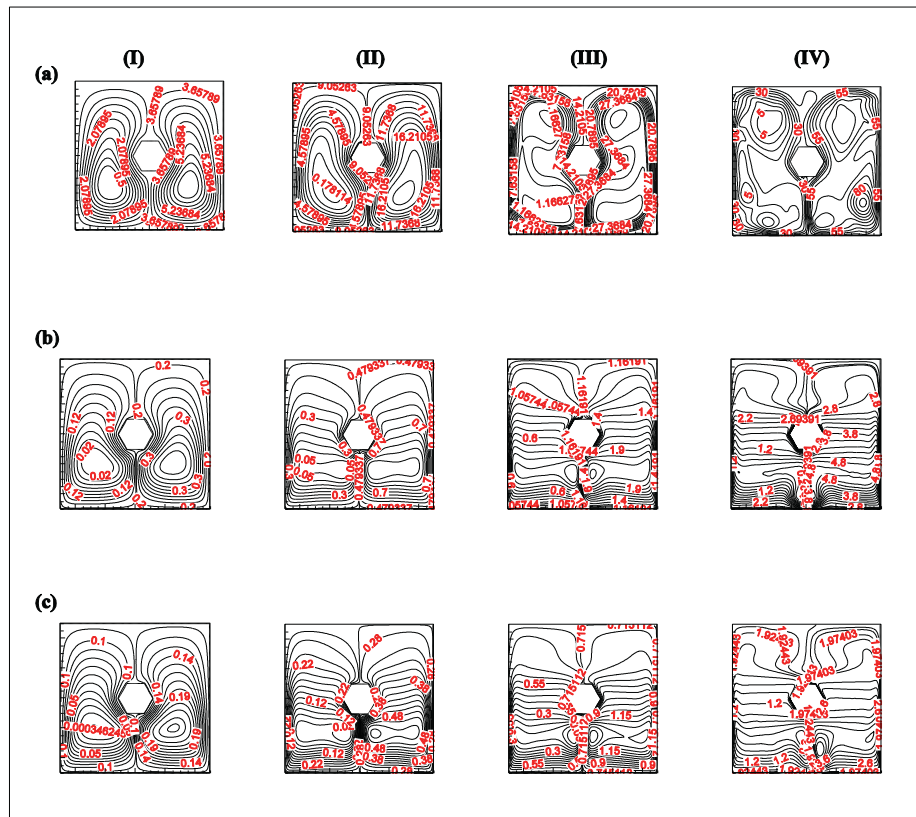


Fig. 6. Variation of streamlines with Grashof ($Gr = \text{I}-10^3$; II- 10^4 ; III- 10^5 ; IV- 10^6) and Prandtl numbers ($Pr = \text{a}-1$; b-50 and c-100) for power law index of $n = 0.5$ for uniform heat flux (UHF) condition.

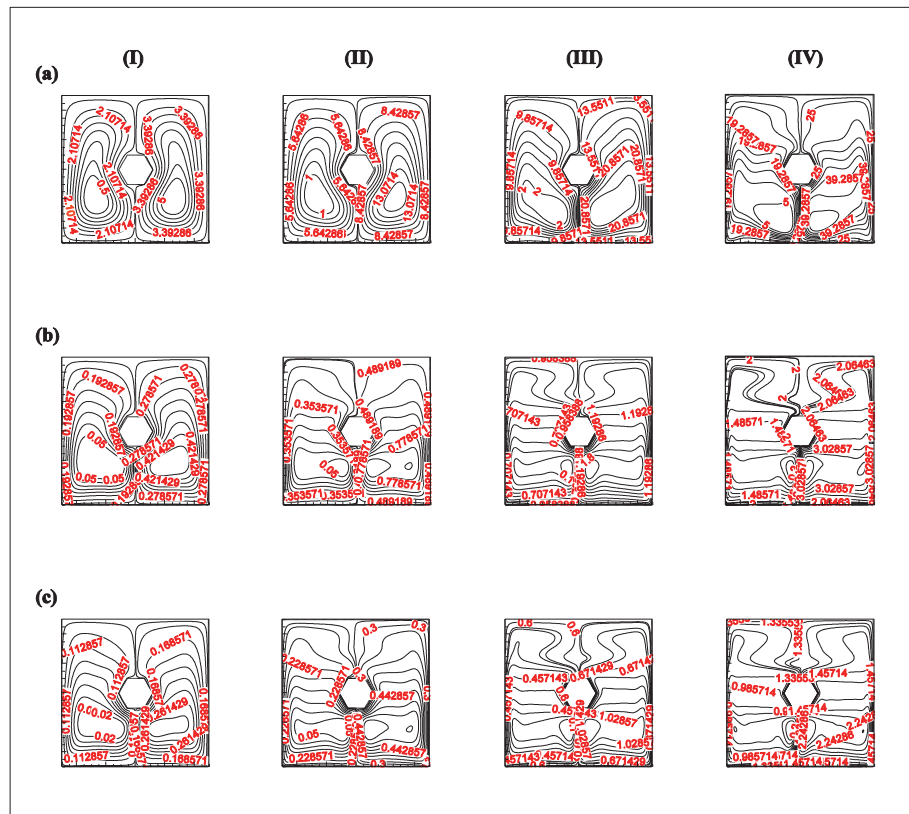


Fig. 7. Variation of streamlines with Grashof ($Gr = \text{I}-10^3$; $\text{II}-10^4$; $\text{III}-10^5$; $\text{IV}-10^6$) and Prandtl numbers ($Pr = \text{a}-1$; $\text{b}-50$ and $\text{c}-100$) for power law index of $n = 1.0$ for uniform heat flux (UHF) condition.

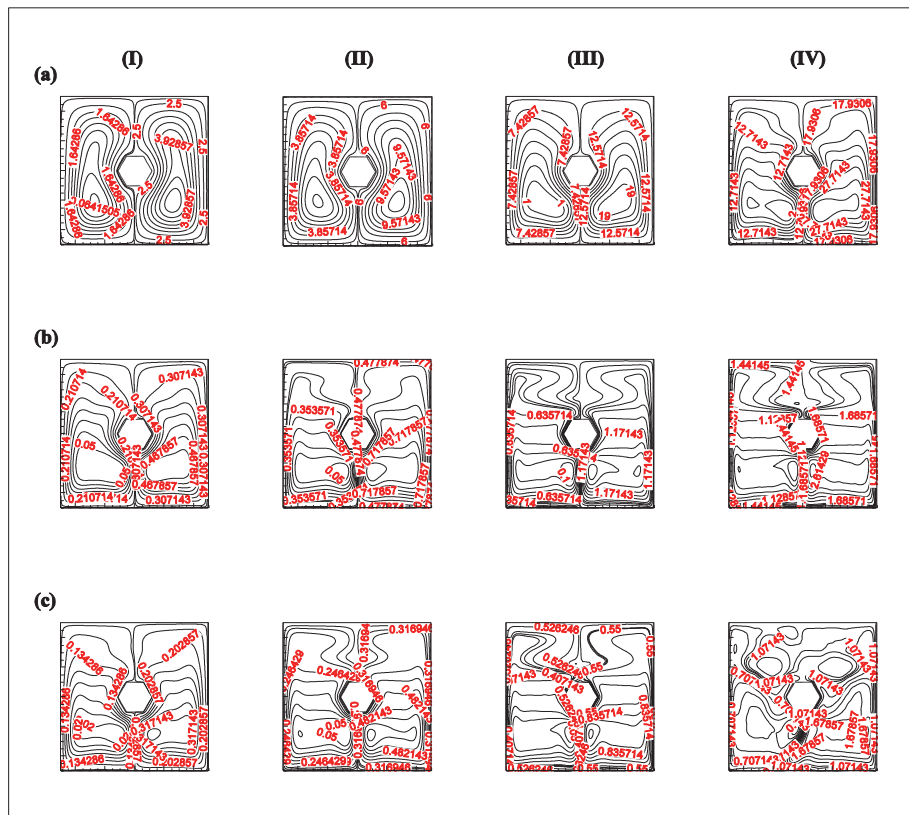


Fig. 8. Variation of streamlines with Grashof ($Gr = \text{I}-10^3$; $\text{II}-10^4$; $\text{III}-10^5$; $\text{IV}-10^6$) and Prandtl numbers ($Pr = \text{a}-1$; $\text{b}-50$ and $\text{c}-100$) for power law index of $n = 1.5$ for uniform heat flux (UHF) condition.

of plume can be noted for $Pr \geq 50$, (Figs 3–5, b and c). It is due to the fact that higher Prandtl number fluids have dominant momentum/hydrodynamic boundary layer than that of thermal one. Simply saying, such fluids are dominated by viscous forces, thus impedes the thermal penetration in fluid. The formation of plume is reported for higher Pr fluids, due to striking of fluid elements completing half circulation over the upper region of hexagonal body (as fluid circulation is mainly between part of hexagonal block facing vertical wall). The size of plume increases with Gr due to prominent circulation fluids for higher Prandtl number cases. The influence of power law index of the plume structure is observed to be nearly insignificant. For low Pr fluid ($Pr = 1$), due to higher fluid movement and lower viscous effects, the formation of plume is not visible for $Gr \leq 10^4$.

Streamline distributions for hexagonal block with uniform heat flux (UHF) case are shown in Figs. 6–8, for power law indexes of $n = 0.5$, 1.0 and 1.5 respectively. A remarkable difference is visible in fluid structure for both thermal conditions. For UHF condition, flow structure is observed to be vertically reverse of that of CWT case. Due to the presence of heat source and gravity effect, the flow circulation originates from the bottom. The convection cells, which are present in upper half of cavity for CWT case, now shift in lower half of cavity. One more difference can be noted that, fluid circulation for UHF case is much predominant than CWT for otherwise same parameters. For higher buoyancy forces,

flow becomes chaotic with enlargement of convection cell. Thus, flow circulation becomes rapid and it limits close to the active walls of cavity. Similar way, formation of plume is also visible for UHF case even for low Prandtl fluids ($Pr = 1$). As the direction of flow is reverse of CWT, the vertical plume formation takes place in downward direction only. The increase in Gr , causes stretching of plume towards lower wall with reduction in its width. Another significant change on the streamline structure can be noted for higher Pr ($Pr \geq 50$) and Gr ($Gr \geq 10^4$), i.e., flow becomes parallel to the horizontal axis with elongation (horizontally) and shifting (towards the lower wall) of convection cells for a given power law index. Such behavior is observed due to accelerated flow due to high intensity buoyancy force. Thus from above illustration, it can be clearly stated that the flow structure is predominantly affected due to type of boundary condition, followed by Prandtl and Grashof number for given power law index.

On the other hand, Figs. 9–14 show temperature profiles in cavity with heated hexagonal block. Fig. 9 delineates isotherm contours for shear thinning fluids ($n = 0.5$) and other similar condition considered herein. At low Grashof number ($Gr = 10^3$), due to conduction dominant heat transfer, isotherm lines are nearly parallel to vertical cold walls. The augmentation in Gr , causes rise in flow circulation, thus making isotherm lines parallel to horizontal walls. Also, crowding of isotherms around hexagonal block was noticed. It can also be seen for higher

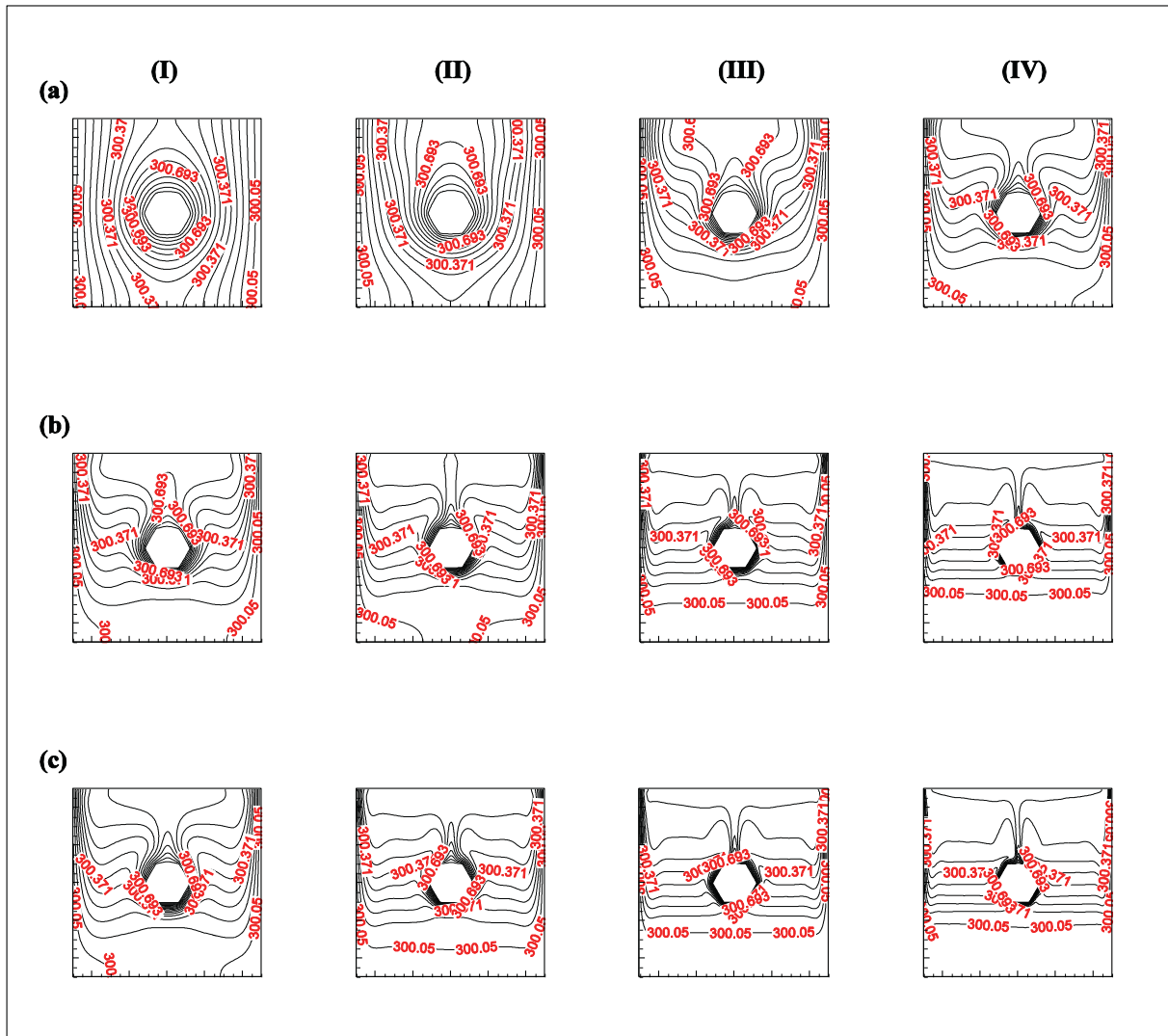


Fig. 9. Variation of isotherms with Grashof ($Gr = \text{I-}10^3$; $\text{II-}10^4$; $\text{III-}10^5$; $\text{IV-}10^6$) and Prandtl numbers ($Pr = \text{a-}1$; $\text{b-}50$ and $\text{c-}100$) for power law index of $n = 0.5$ for constant wall temperature (CWT) condition.

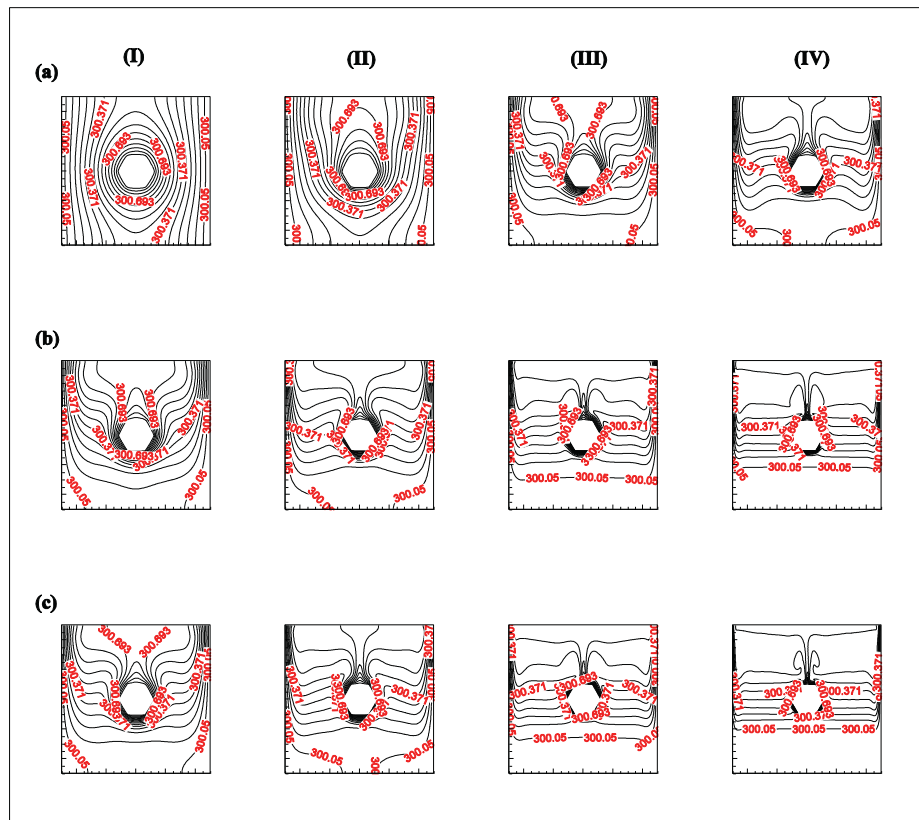


Fig. 10. Variation of isotherms with Grashof ($Gr = I-10^3$; II- 10^4 ; III- 10^5 ; IV- 10^6) and Prandtl numbers ($Pr = a-1$; b-50 and c-100) for power law index of $n = 1.0$ for constant wall temperature (CWT) condition.

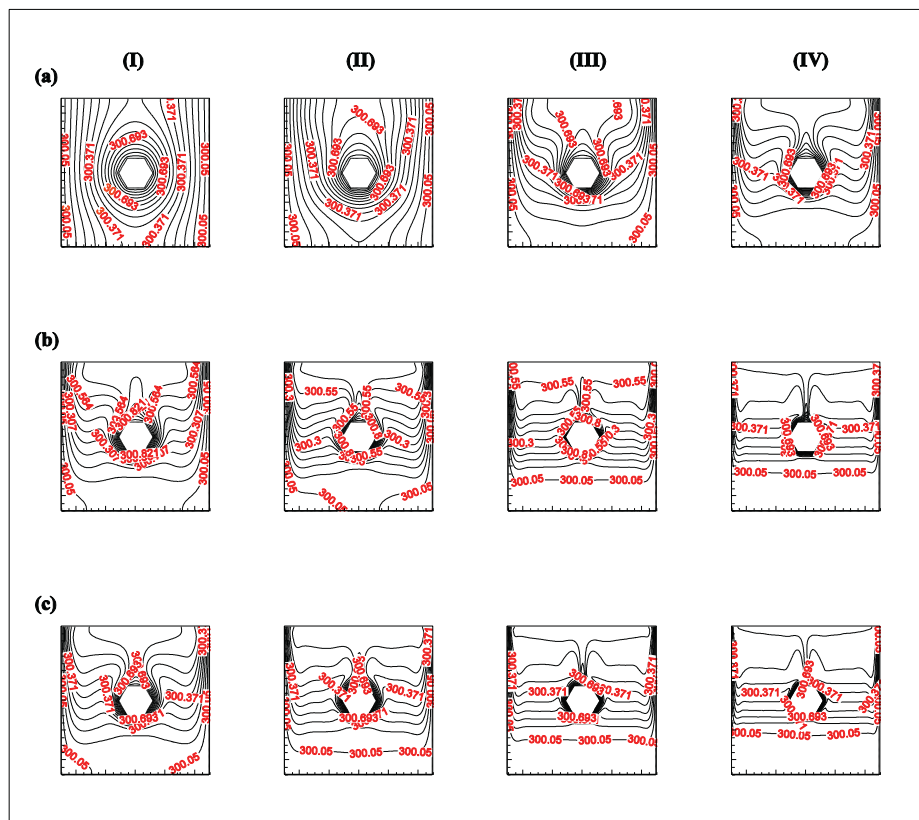


Fig. 11. Variation of isotherms with Grashof ($Gr = I-10^3$; II- 10^4 ; III- 10^5 ; IV- 10^6) and Prandtl numbers ($Pr = a-1$; b-50 and c-100) for power law index of $n = 1.5$ for constant wall temperature (CWT) condition.

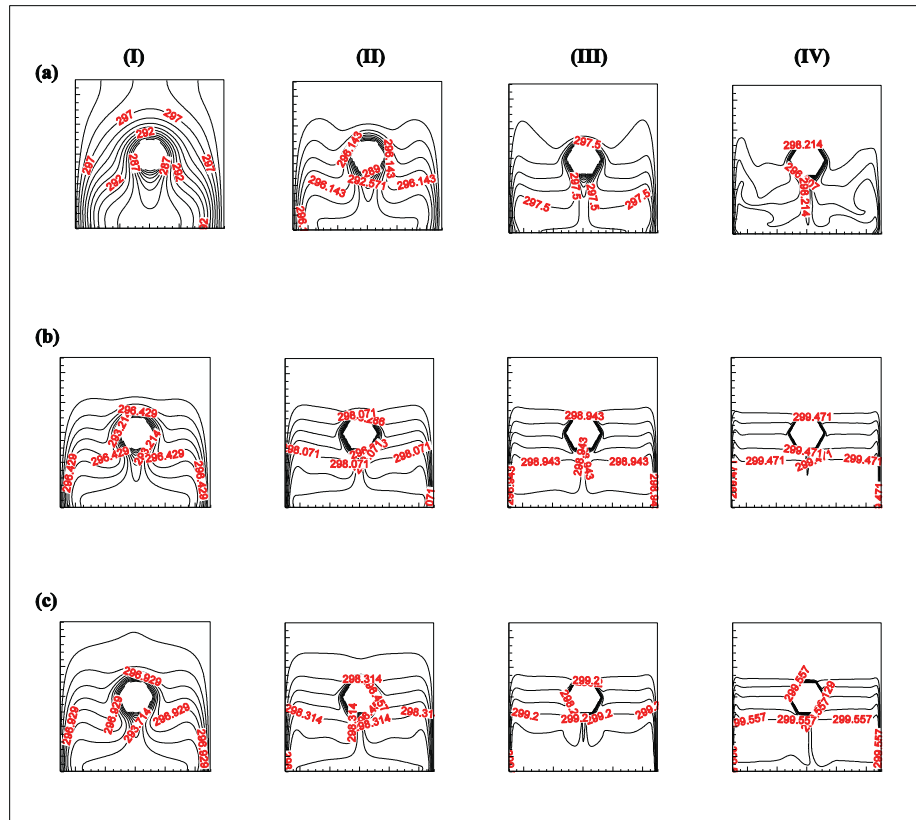


Fig. 12. Variation of isotherms with Grashof ($Gr = I-10^3$; II- 10^4 ; III- 10^5 ; IV- 10^6) and Prandtl numbers ($Pr = a-1$; b-50 and c-100) for power law index of $n = 0.5$ for uniform heat flux (UHF) condition.

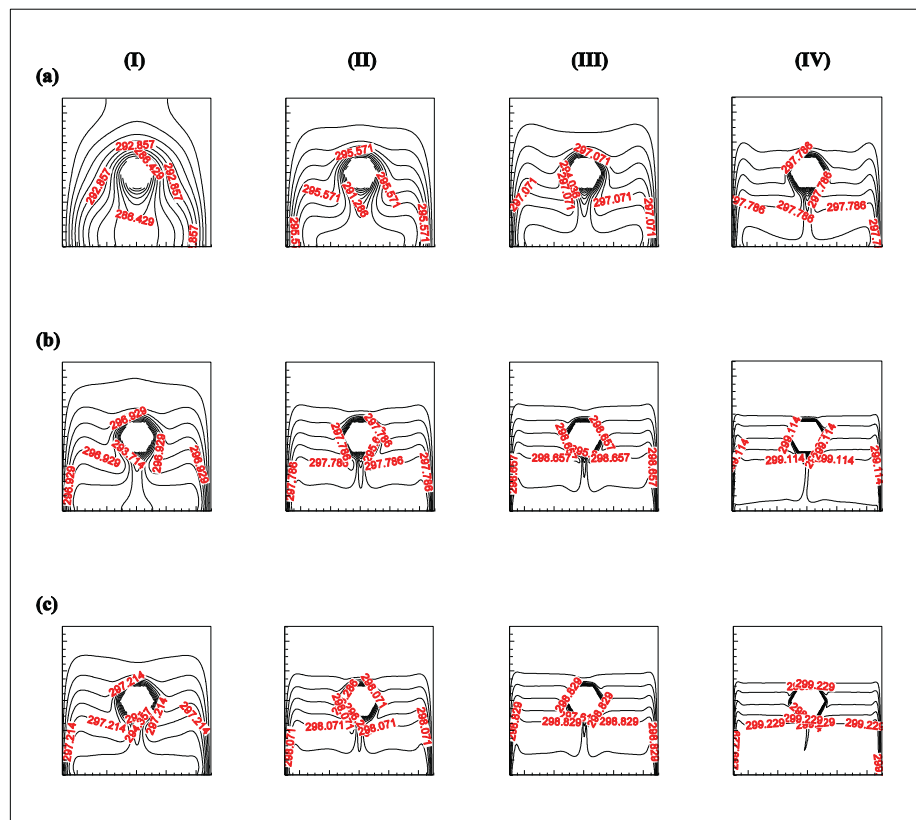


Fig. 13. Variation of isotherms with Grashof ($Gr = I-10^3$; II- 10^4 ; III- 10^5 ; IV- 10^6) and Prandtl numbers ($Pr = a-1$; b-50 and c-100) for power law index of $n = 1.0$ for uniform heat flux (UHF) condition.

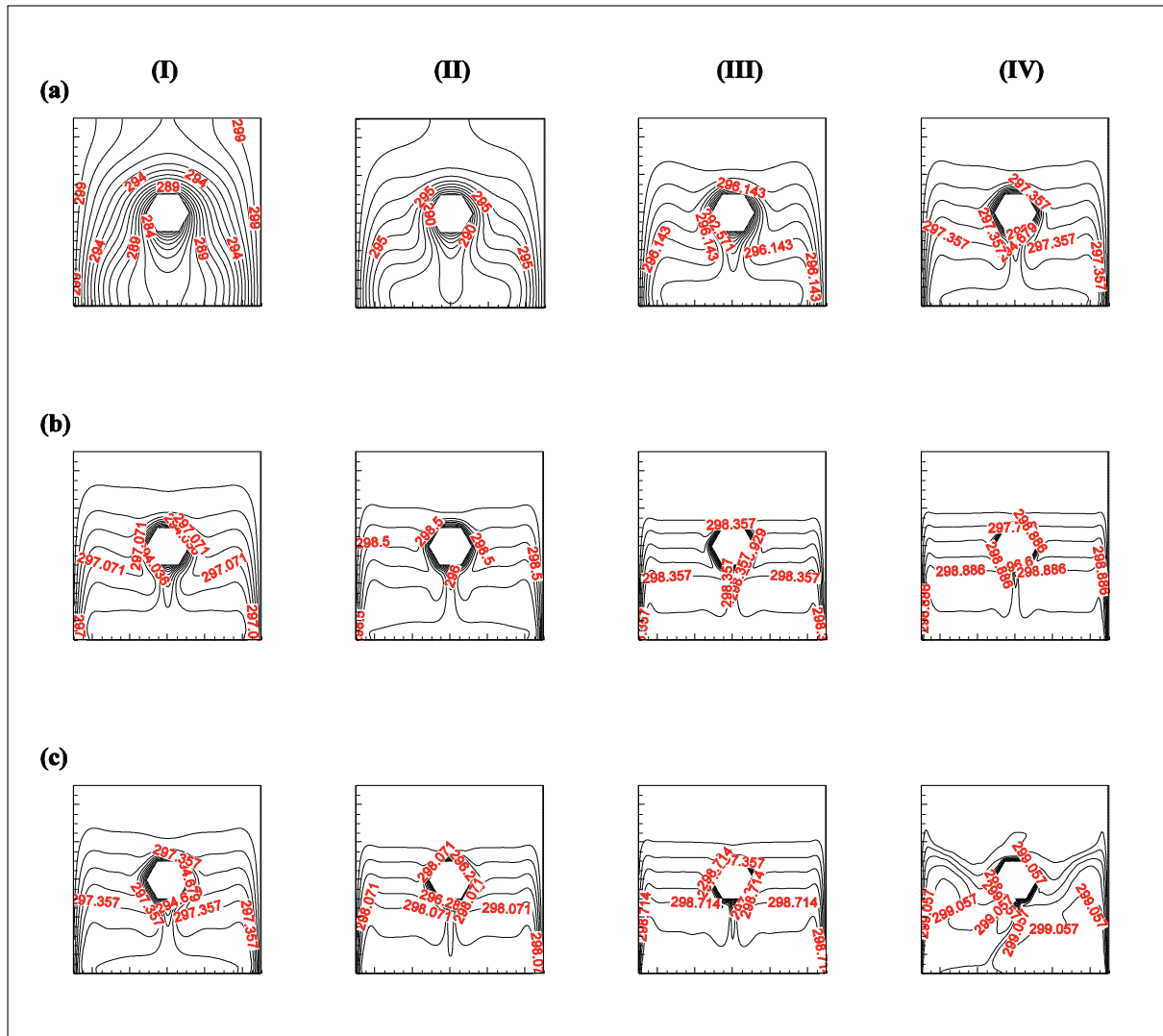


Fig. 14. Variation of isotherms with Grashof ($Gr = \text{I}-10^3$; $\text{II}-10^4$; $\text{III}-10^5$; $\text{IV}-10^6$) and Prandtl numbers ($Pr = \text{a}-1$; $\text{b}-50$ and $\text{c}-100$) for power law index of $n = 1.5$ for uniform heat flux (UHF) condition.

Prandtl number cases ($Pr = \text{a}-1$; $\text{b}-50$ and $\text{c}-100$) that the thickness of isotherm crowding decreases. It is due to the thinning of boundary layer, resulting in higher rate of heat transfer from block to surrounding fluid. Prandtl number increase also restricts the growth of thermal plume. Though, Prandtl number enhancement decreases the thermal penetration capacity of fluid, it enhances the flow circulation with increase in heat transfer rate. Further, shear thinning fluids show higher thermal gradient over the heated body, thus yielding higher heat transfer rate. The change of heated body from shear thinning (Fig. 9) to shear thickening (Fig. 10), the heat transfer rate decreases.

Figs. 12–14 describe the isotherm distribution in cavity with heated block for uniform heat flux (UHF) condition. As per previous discussion of streamlines (Figs. 6–8), similar isotherm pattern are visible, i.e., vertical reverse of CWT condition. The origin and formation of thermal plume takes place in vertically downward direction for UHF and for same range of parameters.

Also, it can be marked that higher crowding of isotherms near block with lower boundary layer as compared to CWT. The dependency of isotherm as well as streamline found to have complex relationship with range of pertinent parameters (Grashof number, Prandtl number, power law index) as well as type of thermal condition. The influence of power law index on flow and isotherm patterns is explored in subsequent section.

5.2. Effect of power law index

The present work has explored the influence of non-Newtonian power law fluids with power law index values, $n = 0.5$, 1 and 1.5 , thus covering Pseudoplastic (shear thinning), Newtonian and dilatant fluids (shear thickening), respectively. The influence of power law index on streamline patterns can be seen from Figs. 3–5 (CWT) and Figs. 6–8 (UHF) with increasing order of n . It can be seen from Figs. 3–8, power law index slightly modify flow pattern, which is visible from the change in recirculation/convection cell zone. The size of convection cell shrinks with increase in power law index indicating reduction in heat transfer rate. Another significant change can be noticed on size of plume formed above the top surface of hexagonal block. Similar effects (as that of streamline patterns) can be noticed on isotherm patterns (Figs. 9–14). The size of thermal plume decreases with n . While less crowding of isotherm patterns near hexagonal block is observed, indicating low temperature gradient, thus, diminishing heat transfer rate due to convection. All in all, it can be clearly concluded from above mentioned discussions that heat transfer rate in cavity is less significantly affected by power law index than Grashof and Prandtl number. The heat transfer characteristics in cavity are delineated in subsequent section.

5.3. Heat transfer characteristics

In this section, dependence of average Nusselt number (Eq. (14)) as well as Colburn factor on natural convection heat transfer (j_{nH}) is explored for range of governing parameters considered herein. Fig. 15 represents the variation of average Nusselt number over the surface of hexagonal block for the considered range of conditions. Plot shows linear increase of average Nu with Grashof numbers, which is due to increased flow due to buoyancy for given power law index. The increase in power law index, i.e., from shear thinning to shear thickening fluid, the density of isotherm clustering reduces, thereby reduces the value of average Nusselt number. Also, for shear thinning fluids, the remarkable deviation in the average Nu values is observed for both thermal conditions. Generally, hexagonal block with heat source or UHF condition yield higher Nu values than that of CWT condition especially for shear thinning fluids. The change of fluid from Newtonian ($n = 1$) and thereby dilatant ($n = 1.5$), and the discrepancy in Nu values for two boundary conditions nearly vanished. Thus it can be proclaimed that the impact of boundary condition over the hexagonal block diminishes for shear-thinning ($n = 0.5$), followed by Newtonian ($n = 1$) and shear-thickening ($n = 1.5$).

In heat transfer studies, the rate of heat transfer is expressed by using average Nusselt number, which is principally a function of Reynolds/Grashof number and Prandtl numbers. All these three parameters (Nu , Re/Gr , Pr) can be combined using a single parameter, called Colburn factor for heat transfer (j_H). For pure forced convection case, it can be expressed as follows:

$$j_H = \frac{\overline{Nu}}{RePr^{1/3}} \quad (18)$$

The Colburn factor for natural convection can be recovered by substituting $V = \sqrt{g\beta\Delta TH}$ in Reynolds number equation. After mathematical rearrangements, the parameter recovered is $\sqrt{Gr}$. Therefore, replacing Re by $\sqrt{Gr}$, the Colburn factor for natural convection heat transfer can be obtained. It is expressed as follows [57]:

$$j_{nH} = \frac{\overline{Nu}}{Gr^{1/2}Pr^{1/3}} \quad (19)$$

Fig. 16 depicts the variation of j_{nH} with Grashof number for different Prandtl numbers and power law indexes. It is known from our previous discussions, higher Nu values are obtained for UHF condition than that of CWT. This effect is seen in variation of j_{nH} also. For given value of power law index, higher j_{nH} values are obtained for UHF case. Moreover, increasing power law index, j_{nH} are predominantly affected at higher Gr values, i.e., $Gr \geq 10^4$. The higher effect of ' n ' on Colburn factor pattern at higher Gr numbers is due to increased buoyancy effect, responsible of density driven flow. Thus, j_{nH} has complex dependence on physical parameters as well as type of boundary condition.

In order to delineate the effect of shape of heated block placed centrally in cavity with otherwise similar boundary as well as parametric conditions, additional computations are performed for standard shapes of circular and square for similar grid structure and size. Comparison is shown in Fig. 17 for Newtonian fluid ($n = 1$), $Pr = 1100$ and both thermal conditions. It is evident from observation of Fig. 17 that higher Nu values are obtained for hexagonal shape, followed by, square and circular. For CWT case, increase in Nu with Gr , is more significant than UHF. Also, as per earlier discussions, it is clear that higher Nu values are obtained for UHF than CWT thermal condition. The variation Nu with Gr and Pr for different shapes is found to be linear/proportional, whereas, some complex dependency between Nu with Gr and Pr for

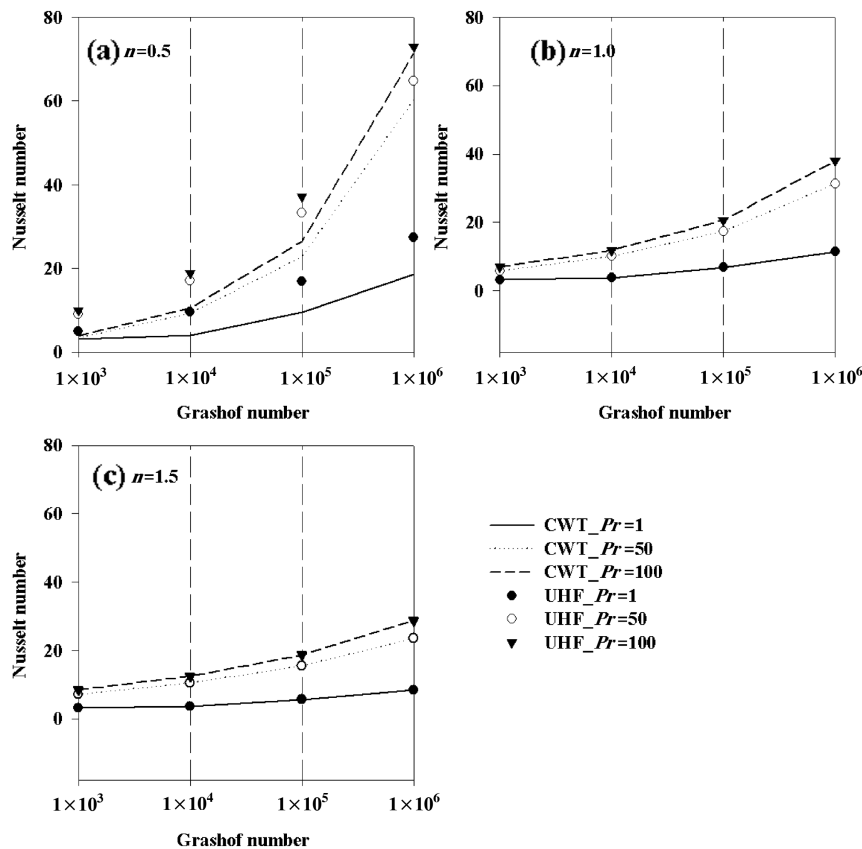


Fig. 15. Variation of $\overline{Nu}$ for different Grashof and Prandtl numbers at given power law index ($n = 0.5, 1$ or 1.5) for thermal conditions of CWT (lines) and UHF (symbols).

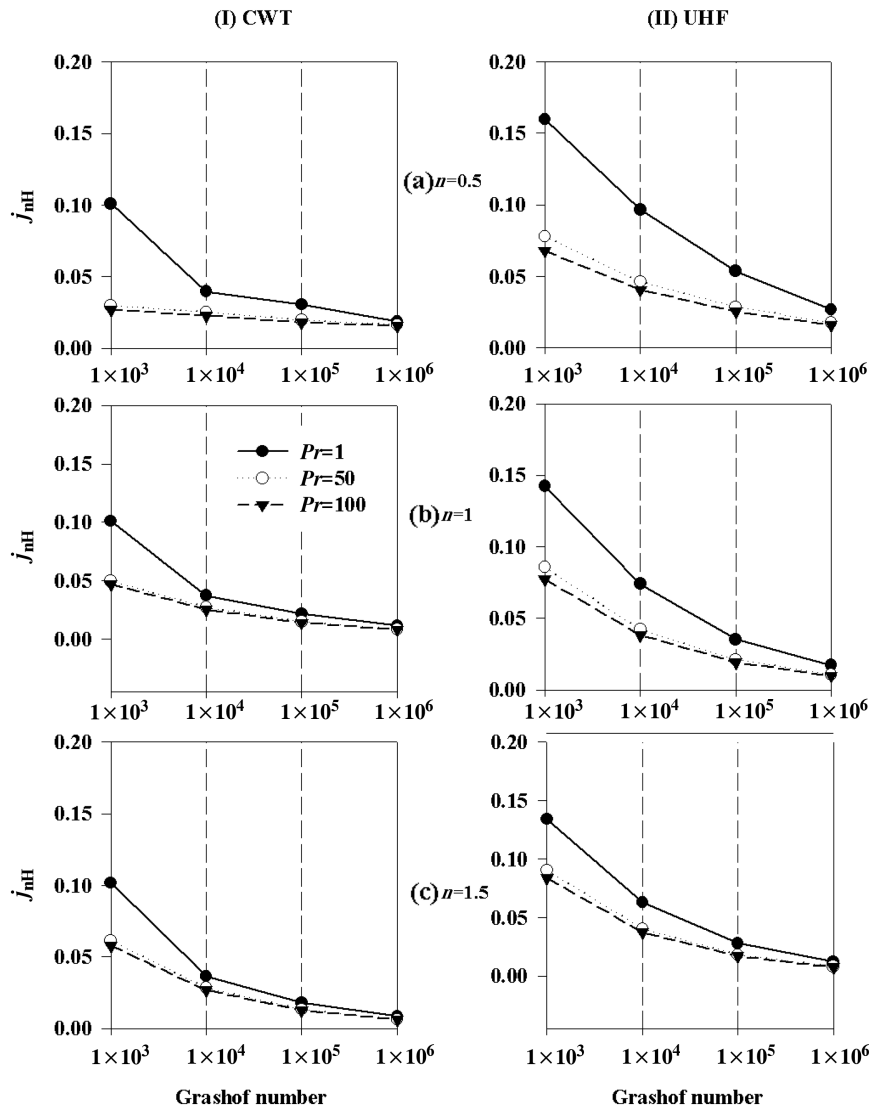


Fig. 16. Dependence of Colburn factor of natural convection (j_{nH}) with Grashof and Prandtl number for different power law indexes ($n = a-0.5$; $b-1$ and $c-1.5$).

different shapes is observed for UHF case. Nu shows very little variation with Gr . Higher Nu values are indicated by square block followed by hexagonal and circular blocks.

5.4. Correlations for Nusselt number and Colburn factor of heat transfer

In heat transfer studies, predictive correlations expressing the functional dependence of the experimental/numerical results are highly valued due to their possible use in the process engineering design and

scientific developments. In the present numerical study, the functional dependence of the average Nusselt number (Nu) of heated hexagonal block and Colburn factor for heat transfer (j_{nH}) on the flow governing parameters, namely, Grashof number ($10^3 \leq Gr \leq 10^6$), Prandtl number ($1 \leq Pr \leq 100$ and power law index ($n = 0.5, 1$ and 1.5), for thermal conditions of CWT and UHF is expressed as follows in general form (basic form for heat transfer correlations):

$$\overline{Nu} \text{ or } j_{nH} = AGr^b Pr^c n^d \quad (20)$$

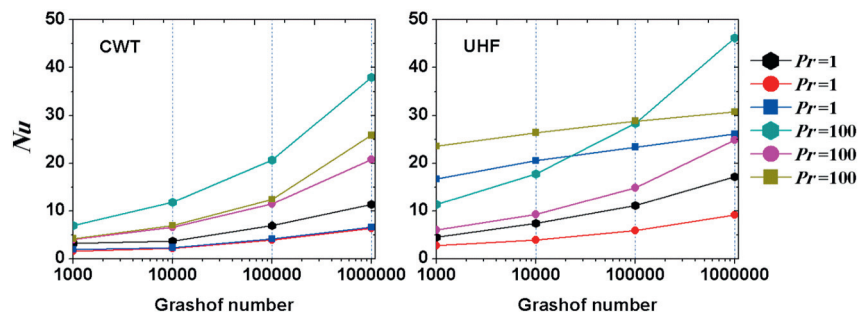


Fig. 17. Effect of shape of heated block (hexagonal, circular and square) on average Nu for range of Pr and Gr for $n = 1$ (Newtonian).

Empirical correlations are developed by using in-house developed MATLAB code for least square curve fitting method. Table 4 presents the developed empirical correlations along with respective R^2 values over the ranges of the parameters considered herein. It can be clearly seen that thermal condition has remarkable influence on exponent of the concerned dimensionless parameters (i.e., Gr , Pr and n). It is observed from analysis of Table 4 that for both thermal conditions, Nusselt number shows linear variation with Gr and Pr ; while it varies in inverse proportion with power law index (n). On the other hand, remarkable change is observed between correlations of j_{nH} for both thermal conditions. In particular, exponent of ' n ' shows opposite behavior for CWT and UHF. Fig. 18 depicts the comparison between the numerical and predicted (by correlations given in Table 4) values of the average Nusselt number. It is clearly visible that the proposed predictive closure correlations predict the average Nusselt number (Nu) as well as Colburn factor (j_{nH})

Table 4

Empirical correlations expressing functional dependence of $\overline{Nu}$ and j_{nH} with Grashof (Gr), Prandtl (Pr) numbers and power law index (n) for CWT and UHF thermal conditions

Thermal condition	Variable	Expression	R^2
CWT	Nusselt number	$\overline{Nu} = \frac{0.41 Gr^{0.25} Pr^{0.22}}{n^{0.17}}$	0.99
	Colburn factor	$j_{nH} = \frac{0.66 n^{0.17}}{Gr^{0.26} Pr^{0.12}}$	0.98
UHF	Nusselt number	$\overline{Nu} = \frac{1.13 Gr^{0.20} Pr^{0.18}}{n^{0.34}}$	0.96
	Colburn factor	$j_{nH} = \frac{1.06}{Gr^{0.29} Pr^{0.13} n^{0.33}}$	0.97

values within an acceptable level of deviations ($\pm 10\%$) from the computed values.

6. Concluding Remarks

The numerical investigation of two dimensional, steady, laminar natural convection for power law fluids in square enclosure with built-in hexagonal heated block for two limiting thermal conditions of constant wall temperature (CWT) and uniform heat flux (UHF) is carried out by using finite volume method with SIMPLE algorithm and QUICK scheme for discretization of convection terms. Natural convection characteristics are analyzed for wide range of pertinent flow governing parameters such as, Grashof number ($10^3 \leq Gr \leq 10^6$) corresponding to laminar flow, Prandtl number ($1 \leq Pr \leq 100$) and power law index ($0.5 \leq n \leq 1.5$). For numerical results, optimum grid size is chosen by carrying out systematic grid independence study. The numerical validation of present work with literature has shown excellent agreement. For considered parameter range, hexagonal block with uniform heat flux (UHF) showed higher heat transfer rate. The dependence of average Nusselt number is more significantly affected by Prandtl number, followed by Grashof number, type of thermal boundary condition and lastly power law index. Further, rate of heat transfer diminishes from shear-thinning to Newtonian followed by shear thickening. The Colburn factor for natural convection heat transfer (j_{nH}) is also determined. The UHF boundary condition shows higher values of j_{nH} than CWT condition, proclaiming higher heat transfer rate for UHF case. In the end, numerical results are summarized by means of closure relationship revealing dependency of Nusselt number and Colburn factor with considered flow governing parameters.

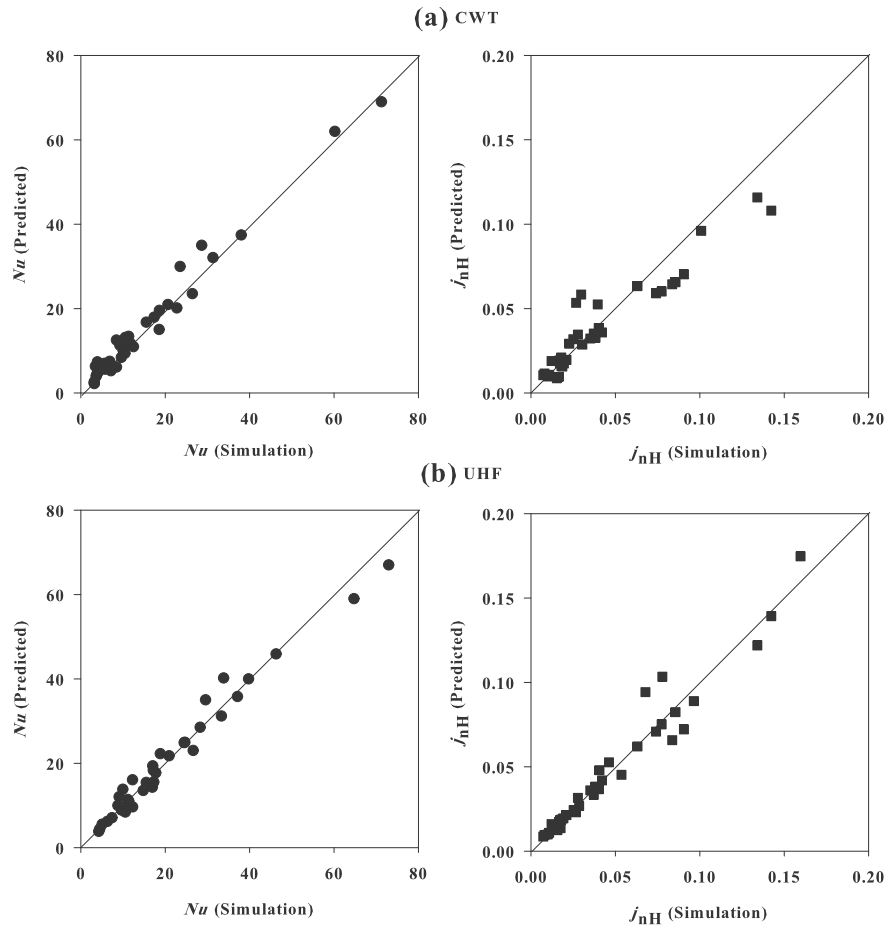


Fig. 18. Parity plot between simulated and predicted (Eq. (20) and Table 4) values of average Nusselt number.

Nomenclature

A_R	aspect ratio
b	edge of equilateral hexagonal block, m
C_p	heat capacity, $J \cdot kg^{-1} \cdot K^{-1}$
CWT	constant wall temperature, K
Da	Darcy number
Gr	Grashof number
H	height of cavity, m
h	heat transfer coefficient, $W \cdot m^{-2} \cdot K^{-1}$
k	thermal conductivity, $W \cdot m^{-1} \cdot K^{-1}$
L	length of cavity, m
Ma	Mach number
Nu	local Nusselt number
$\overline{Nu}$	average Nusselt number
P	pressure, $N \cdot m^{-2}$
Pe	Peclet number
Pr	Prandtl number
q_w	heat flux, $W \cdot m^{-2}$
Ra	Rayleigh number
Re	Reynolds number
Ri	Richardson number
T	temperature
T_c	ambient temperature
T_{ref}	reference temperature, K
T_w	surface temperature of triangular block
T^*	dimensional temperature, K
ΔT	temperature difference, K
UHF	uniform heat flux, $W \cdot m^{-2}$
u_x, u_y	velocity components, $m^2 \cdot s^{-1}$
V	characteristics velocity, $m^2 \cdot s^{-1}$
x, y	co-ordinates, m
α	thermal diffusivity, $m^2 \cdot s^{-1}$
β	coefficient of thermal expansion, K^{-1}
μ	dynamic viscosity, $N \cdot s \cdot m^{-2}$
ν	kinematic viscosity, $m^2 \cdot s^{-1}$
ρ	density, $kg \cdot m^{-3}$
$\bar{\rho}$	average density determined at T_{ref} , K
ϕ	vorticity, $m^2 \cdot s^{-1}$
Ψ	stream function, $m^2 \cdot s^{-1}$
Ω	angular rotational speed, $r \cdot min^{-1}$

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